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Effect of exothermic reactions downstream of the C–J plane on detonation stability

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Abstract—The effect of exothermic reactions on the gasdynamics of one-dimensional flow in a nonsteady "burn-down" zone which follows a plane detonation wave is considered. The dynamics of secondary compression waves downstream of the *C–J* plane is calculated. Depending on the heat release kinetics in the "burn-down" zone the secondary compression wave is either transformed into a shock wave which overtakes the primary detonation wave and changes its velocity abruptly, or creates a subsonic flow region adjacent to the *C–J* plane and results in the smooth acceleration of the detonation wave. The dwell times of low velocity *C–J* detonations are estimated for long and short acting initiators. The results of the analysis are applicable at least qualitatively to a wide variety of nonideal explosive systems.

Introduction

THE DETONATION velocity in many explosive systems under certain conditions turns out to be substantially lower than the ideal thermodynamic value. The apparent *C–J* plane (or surface) in so called nonideal detonation waves normally corresponds to an intermediate stage of the reaction, i.e. some part of the chemical energy stored in the explosive system is being released in the rearfaction zone. The mechanism of nonideal detonations in the case when reacting material is allowed to expand laterally has been discussed in numerous papers (i.e. N. M. Kuznetsov, 1968; B. S. Ermolaev *et al.*, 1976; R. F. Chaiken and J. C. Edwards, 1976), and its main features are well known. There is another type of nonideal detonations, those are the waves with nonmonotonous heat release (N. M. Kuznetsov, 1968). One-dimensional or quasi-onedimensional detonation waves with consecutive fast and slow reaction stages are observed in all types of the explosive systems. Low velocity detonation (LVD) regimes in condensed explosives have the ignition in hot spots and burning of the explosive material before the moment of its fragmentation as a fast reaction stage and burn out of the dispersed HE as a slow reaction stage. The slow reaction in heterogeneous explosives is burning of less reactive explosive or fuel component (i.e. Metall). The region of slow heat evolution exists also in detonation waves in sprays (burning of remnants of droplets) and in gaseous systems with complicated kinetics of the reaction (e.g. in hydrocarbon-oxygen mixtures). The effective heating rate of gases downstream of the detonation front in the case of nonmonotonous heat release should comprise heat transfer between phases and heat losses as well and thus under certain conditions it can change its sign in some regions of a flow.

Two questions arise

What are the conditions under which the detonation wave with partial heat release propagates steadily in a tube with rigid (or slightly deformable) walls, and what kind of transient phenomena are to be expected if some part of chemical energy is released downstream of the *C–J* plane in the zone of a rearfaction wave.

According to experimental observations (e.g. A. N. Dremin *et al.*, 1970) the heat release in an extended zone downstream of the lead shock wave results either in continuous acceleration of the detonation wave or in the generation of a secondary shock wave in the expanding and reacting two– or one–phase flow. The secondary shock wave eventually overtakes the lead shock. Steady-state regimes of nonideal one-dimensional detonations have been observed in charges of limited length and diameter. The stability of onedimensional LVD which are characterized by a very small portion of the total energy released upstream of the *C–J* plane has to be studied in more details. N. M. Kuznetsov (1968) has shown that nonideal detonations are stable with respect to small linear disturbances. However his analysis does not account for the nonlinear effects (e.g. formation of secondary shock waves) downstream of the *C–J* plane (in burn down zone). Thus the analysis of the LVD stability should be completed with the consideration of a flow in the nonisentropic rearfaction wave and of an interaction of nonlinear disturbances generated in this flow with the flow upstream of the *C–J* plane.

This paper gives the analysis of gasdynamic phenomena in one-dimensional flow of a reacting fluid downstream of the *C–J* plane. Using analytical solutions the dynamics of secondary compression waves formation and their interaction with the primary detonation wave is considered. An assumption is made that the detonation wave always starts as a *C–J* wave with partial heat release, and the rest of the chemical energy is evolved at a rate comparable with the rate of flow parameters change in the rearfaction wave which follows the detonation wave.

Gasdynamics of one-dimensional flow in a burn-down zone of a detonation wave. Let us consider the flow created by a movement of a plane piston in a quiescent homogeneous fluid which is capable to react exothermically. The piston trajectory is chosen in such a way that:

(1) the shock wave generated by the piston propagates at a constant velocity at least for the period of time less than a certain value t_{CJ},

(2) the shock wave parameters are high enough to initiate the fast stage of the chemical reaction,

(3) at $t = t_{CJ}$ the fast stage of the reaction is completed (heat release rate turns out to be equal to zero), and there appears a sonic (with respect to the detonation front) plane at the piston surface,

(4) further piston motion is accompanied with a slow heat release in the burn-down zone.

The absence of shock waves in the burn-down zone would mean that for $t > t_{CJ}$ the wave configuration consisting of a lead shock wave and of a zone of the fast reaction is propagating at a constant velocity D_{CJ} and can be considered as a *CJ* detonation with a heat release determined by the fast stage of the reaction. If $D \neq D_{CJ}$ then a transient regime arises which finally results in the unsupported detonation wave propagating at the velocity D_{CJ}. Some aspects of these regimes were discussed earlier (A. N. Dremin *et al.*, 1970; B. S. Ermolaev *et al.* 1976), however the transient phenomena of this type are not the subject of concern in the analysis given below.

The governing equations for the flow can be written as follows:

$$\begin{aligned} &\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0; \quad \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} \\ &\frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \\ &\frac{d}{dt}\ln p/\rho^{\gamma} = \frac{\rho(\gamma-1)}{p}\frac{dQ}{dt}. \end{aligned} \tag{1}$$

The equation of state is

$$E = p/(\gamma - 1)\rho; \quad \gamma = \text{Const.} \tag{2}$$

The boundary conditions at the front of strong shock and at the piston are same as those used by H. M. Sternberg (1970) in the case of a plane flow. It is convenient to characterize the flow parameters distribution downstream of the lead shock using coordinate ξ equal to time measured along the Lagrangian particle path: $\xi = t - t_1$, where $t_1 = x_s/D$ is Lagrangian label for a particle path, $\xi = 0$ at the shock front and $d/dt = \partial/\partial\xi$.

The flow dynamics is evidently strongly dependent on Q as a function of time. Here Q is an effective heat release which includes heat production due to chemical reactions and heat losses. Only the waves with $\partial Q/\partial\xi = 0$ at least in one point of the flow where $Q > 0$ but $Q \neq Q_\infty$ are considered below. This point corresponds to the completion of the first (fast) stage of the reaction (at $\xi = t_{CJ}$). It can be shown that the solution $D = D_{CJ}$ where D_{CJ} is calculated using equations

$$\frac{\partial Q}{\partial \xi} = 0, \quad M_{CJ} = 1 \tag{3}$$

is the only steady-state solution for $0 \leq \xi \leq t_{CJ}$ where t_{CJ} is defined for the first maximum on the $Q(\xi)$ curve.

Before to go to the analysis of what happens if the energy starts to evolve again (with some time delay) at $\xi > t_{CJ}$, i.e. in the burndown zone of the wave propagating at the velocity D_{CJ}, it is necessary to characterize the behavior of the flow parameters in the vicinity of the *CJ* plane.

Since the *CJ* point is simultaneously the head of the rearfaction wave the derivatives $\partial p/\partial\xi$, $\partial u/\partial\xi$ and $\partial\rho/\partial\xi$ have discontinuity at this point. However p, u and ρ themselves are continuous functions of time and space coordinate at this point. The derivative $(\partial Q/\partial\xi)_-$ upstream of the *CJ* plane is equal to 0 (see eqn 3).

It is a function of time and flow parameters rather than their derivatives therefore

$$(\partial Q/\partial \xi)_+ = 0 \tag{4}$$

from the burn-down zone side, i.e. heat release rate must be continuous at the CJ point. This conclusion although evident is important for the correct analysis of gasdynamics in the burn-down zone. Equation (4) together with $M_{CJ}^2 = 1$ defines the boundary conditions at $\xi = t_{CJ}$ for the burn-down zone.

It is not difficult to show that a steady nonideal detonation wave propagating at the velocity D_{CJ}, which corresponds to a partial heat release, can not be realized in any piston flow if the net energy evolved both in the detonation and burn-down zones exceeds the energy evolved before t_{CJ} only, i.e. if $Q_2 = Q_\infty - Q_{CJ}$ is positive. Let us consider for this purpose a selfsimilar solution which is valid in the both zones ($0 \leqslant \xi \leqslant t_{CJ}$ and $\xi > t_{CJ}$). Since an assumption has been made that $D = D_{CJ} =$ Const all the parameters for $0 \leqslant \xi \leqslant t_{CJ}$ are functions of ξ only. The possibility of analytical continuation of the well known solution for the steady detonation zone through the CJ plane into the burn-down zone using the appropriate choice of the piston trajectory has been shown by Sternberg (1970). Integrating the conservation eqns (1) one gets for the case of a strong detonation wave ($c_0{}^2 \gg Q_{CJ}$):

$$\begin{aligned} p &= p_{CJ}[1 \pm \sqrt{(1 - 2(\gamma^2 - 1)Q(\xi)/D^2)}] \\ M &= \sqrt{[(\rho_0 D^2/p - 1)/\gamma]}. \end{aligned} \tag{5}$$

The piston trajectory is given by the equation

$$x_p = u_{CJ} \int_0^t [1 \pm \sqrt{(1 - 2(\gamma^2 - 1)Q(\xi)/D^2)}]\, d\xi \tag{6}$$

$Q(\xi)$ is calculated using the kinetic equation

$$dQ/d\xi = F(\xi, p, \rho, E) \tag{7}$$

with the initial condition $\xi = 0$; $Q = 0$.

The sign plus should be taken in the expressions for p and x_p in the subsonic detonation zone, and minus corresponds to the supersonic burn-down zone. There are no other restrictions for the value of F in the burn-down zone besides $F(\xi_{CJ}) = 0$.

The physical meaning of this solution is based on the fact that at large enough times any isentropical one-dimensional rearfaction wave becomes more and more like a steady uniform flow. Hence the flow pattern nearby the nonideal detonation wave asymptotically approaches the selfsimilar solution dependant on ξ only. Indeed expanding the parameters p, ρ and u in eqn (1) in Taylor series nearby $\xi = t_{CJ+}$ and then solving the resulting ordinary differential equations one gets the law according to which the pressure gradient reaches its selfsimilar value $(\mathrm{d}p/\mathrm{d}\xi)_{ss}$ at $\xi = t_{CJ}$:

$$\left.\frac{\partial p}{\partial \xi}\right|_+ = \left.\frac{\mathrm{d}p}{\mathrm{d}\xi}\right|_- \operatorname{cth}\left[\frac{(\gamma+1)^2}{2\rho_0 D^2}\left(-\left.\frac{\mathrm{d}p}{\mathrm{d}\xi}\right|_-\right)(t+t_0)\right] \to \left.\frac{\mathrm{d}p}{\mathrm{d}\xi}\right|_- \text{ when } t \to \infty \text{ and } \left.\frac{\mathrm{d}^2 Q}{\mathrm{d}\xi^2}\right|_- < 0. \tag{8}$$

Here t_0 is the integration constant characterizing the value of $\partial p/\partial\xi|_+$ at the moment of the CJ plane origination, $t_0 = 0$ if the predetonation column length is equal to 0.

The case of an endothermic process in the burn-down zone ($Q_2 = Q_\infty - Q_{CJ} <$ 0, see Fig. 1a) gives a truly steady solution: the integral curve $p(\xi)$ (Fig. 1b, curve 2) which corresponds to $D = D_{CJ} = \sqrt{(2(\gamma^2 - 1)Q_{CJ})}$ (here Q_{CJ} is the largest maximum value of Q) satisfies the boundary conditions at the shock front and at $\xi = t_{CJ}$. The flow downstream of the CJ point does not change the parameters in the detonation zone. The piston trajectory for this case is shown in Fig. 1(c). All the values of D less than D_{CJ} (curves 3 and 4 of Fig. 1b) result in flow choking and generation of secondary shock waves in the burn-down zone (H. M. Sternberg, 1970; B. S. Ermolaev *et al.*, 1976). The waves with $D > D_{CJ}$ represent supported detonations and are not of interest for present analysis (curve 1, Fig. 1b).

The case of an exothermic process in the burn-down zone ($Q_2 > 0$) (Fig. 2a) is more interesting from the practical point of view. The integral curve for the CJ wave propagating at a velocity $D_{CJ,1} \sim \sqrt{(Q_{\max,1})}$ is marked as a curve 1 in Fig. 2(b). The pressure in the supersonic burn-down zone starts to rise when $\partial Q/\partial\xi$ becomes positive. This pressure rise results in flow choking (when $p \to p_{CJ}$, $\partial p/\partial\xi \to \infty$, $M \to 1$ but $\partial Q/\partial\xi \neq 0$) at the piston when $Q(\xi) = Q(\xi_2)$ becomes equal to $Q_{\max,1}$ again. The secondary shock wave generated at the piston (Fig. 2c) at $t = t_{cr}$ overtakes the primary detonation wave and change its velocity abruptly.

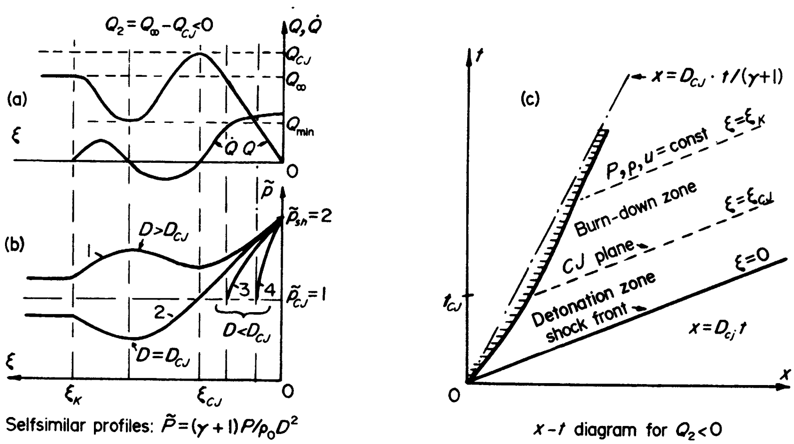

Fig. 1. Selfsimilar solution for $Q_2 < 0$; (a) Q and $\dot{Q}$ profiles (b) dimensionless pressure profile, and (c) $x - t$ diagram. Curve 1: $D > D_{CJ}$; 2: $D = D_{CJ}$; 3 and 4: $D < D_{CJ}$.

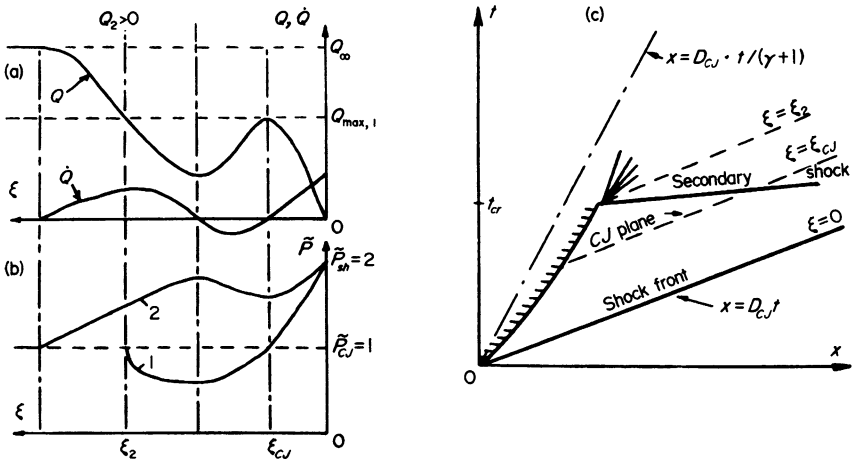

Fig. 2. Selfsimilar solution for $Q_2>0$; (a) Q and $\dot{Q}$ profiles, (b) dimensionless pressure profile, and (c) $x-t$ wave diagram. Curve 1, $D=\sqrt{[2(\gamma^2-1)Q_{max\,1}]}$; Curve 2, $D=\sqrt{[2(\gamma^2-1)Q_\infty]}$.

Number 2 in Fig. 2(b) denotes the integral curve for the wave propagating at the velocity $D_{CJ,2} \sim \sqrt{(Q_\infty)}$, corresponding to the largest maximum of $Q(\xi)$.

This selfsimilar asymptotic analysis shows that the detonation wave with partial reaction completeness is truly stable only in the case when the kinetics of heat release and losses provides the energy production profile at a very long time after initiation having the largest maximum at the first stage of the reaction, i.e. $Q_2 \leq 0$. Otherwise ($Q_2>0$) the wave will propagate at a constant low velocity only for a limited period of time.

Dynamics of secondary shock waves formation in the burn-down zone

In order to simulate the unsupported detonation wave propagation from the closed end in a rigid tube one has to admit that the piston—which initiated the detonation wave and the fast stage of the reaction—stops at $t=t_{CJ}$ creating a rearfaction wave, i.e. a nonsteady burn-down zone.

The conservation equations in the nonsteady burn-down zone can be integrated analytically using the perturbation technique if the energy evolved in this zone exceeds Q_{CJ} only slightly, that is if $Q_2/Q_{CJ}=\epsilon^2 \ll 1$. As shown by J. B. Bdzil (1976) the condition $\epsilon^2 \ll 1$ allows to reduce the set of eqns (1) to only one equation:

$$\frac{\partial w_1}{\partial \tau}+w_1\frac{\partial w_1}{\partial y}=-\frac{1}{2\gamma^2}\frac{\partial q}{\partial y} \tag{9}$$

where

$$\tau=c_{CJ}t/\epsilon l;\; y=\frac{2(D_{CJ}t-x)}{(\gamma+1)l};\; w_1=\frac{\gamma+1}{\gamma}\left(\frac{D_{CJ}}{\gamma+1}-u\right)/\epsilon D_{CJ};\; q=(Q-Q_{CJ})/\epsilon^2 Q_{CJ}.$$

This equation gives the asymptotic solution which is valid for the entire region restricted by the CJ plane trajectory (which is assumed to be a straight line $x = D_{CJ}t$) and by the quiescent piston ($x = 0$). The boundary conditions at the CJ plane are

$$y = 0; \quad w_1 = 0; \quad q = 0; \quad \partial q/\partial y = 0. \tag{10}$$

The solution obtained by Bdzil hardly could represent any practically interesting case of nonideal detonation because he used the heat release function $q(y)$ having discontinuous derivative at the initial CJ plane, i.e. $\partial Q/\partial\xi|_+ \sim \partial q/\partial y|_+ \neq 0$. That is why the detonation wave in his solution starts to accelerate without any delay at $t = 0$.

If $\partial q/\partial y \equiv 0$ the solution of eqn (9) represents the classical isentropic rarefaction wave attached to the CJ plane of an ideal detonation wave.

In general q depends on τ and y but for convenience we assume it independent on τ (otherwise the eqn (9) can be solved only numerically). This assumption means that the heating rate is not affected by the change of flow parameters in the burn-down zone. The purpose of the present paper is to elucidate the main features of the process, therefore a simple expression for $q(y)$ has been chosen:

$$q(y) = \gamma^2 \begin{cases} -A[1 + \mathrm{sh}\, n(y - y_0)]^2/\mathrm{ch}^2\, n(y - y_0); & 0 \leqslant y \leqslant 2y_0 \\ -2A + B\, \mathrm{th}^2\, m(y - 2y_0) & y > 2y_0 \end{cases} \tag{11}$$

where $y_0 = 1/n$ arsh (1). Parameters A and B determine the amount of the heat lost and/or gained in the burn-down zone, n and m characterize the rate of heat dissipation and evolution. At the CJ plane ($y = 0$) eqn (11) gives $\mathrm{d}q/\mathrm{d}y = 0$, hence the detonation zone (upstream of the CJ plane) will be stable at least for small time interval τ. Figure 3 demonstrates the typical profiles of $(Q - Q_{CJ})/Q_{CJ} = q(y)\epsilon^2$ for the case when $D_{CJ} = 8$ mm/μs, $\gamma = 3$ and $1 = 5$ mm.

The solution of eqn (9) for $0 \leqslant y \leqslant 2y_0$ and $A > 0$ is:

$$\tau = \frac{1}{n\sqrt{(AZ)}}\,\mathrm{arsh}\,\frac{1 + Z\,\mathrm{sh}\, n(y - y_0)}{\sqrt{(Z^2 - 1)}} \tag{12}$$

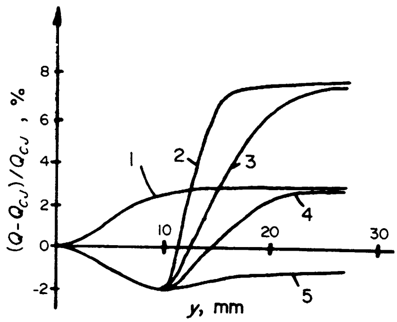

Fig. 3. Typical heat release profiles in the burn-down zone for different values of the parameters A, B, n and m (see eqn 11).

where $Z = w_1^2/A - 2\,\mathrm{sh}\,n(y-y_0)/\mathrm{ch}^2\,n(y-y_0)$. For $y > 2y_0$

$$\tau - V = \begin{cases} \dfrac{1}{m\sqrt{(-S)}}\,\mathrm{arsh}\,\dfrac{\sqrt{(-S)}\,\mathrm{sh}\,m(y-y_02)}{\sqrt{(B-S)}} & S \leqslant 0 \\[2ex] \dfrac{1}{m\sqrt{(S)}}\,\mathrm{arsin}\,\dfrac{\sqrt{(S)}\,\mathrm{sh}\,m(y-y_02)}{\sqrt{(B-S)}} & S > 0 \end{cases} \tag{13}$$

where

$$S = B[1 - \mathrm{th}^2\,m(y-y_02)] - w_1^2; \qquad V = \frac{1}{n\sqrt{(B-A-S)}}\,\mathrm{arsh}\,\frac{1}{\sqrt{[1-2A/(B-S)]}}.$$

First we consider the net exothermic process in the burn-down zone (curve 1 in Fig. 3). In this case $A = 0$, $y_0 = 0$ and $V = 0$. For small y one gets from eqn (13)

$$w_1(y \to 0) = m\sqrt{(B)}\,y\,ctg\,m\sqrt{(B)}\,\tau. \tag{14}$$

The gradient of the particle velocity at the CJ plane can be expressed as follows:

$$\left.\frac{\partial w_1}{\partial y}\right|_{y\to 0} = m\sqrt{(B)}\,ctg\,m\sqrt{(B)}\,\tau \sim -\left.\frac{\partial u}{\partial y}\right|_{y\to 0}. \tag{15}$$

Thus the rearfaction wave disappears and the flow in the vicinity of CJ plane becomes subsonic with respect to the front ($u < u_{CJ}$) when $\tau = \tau_{cr}/2 = \pi/2m\sqrt{(B)}$. The secondary shock wave arises at $\tau = \tau_{cr}$ (when $\partial u/\partial y \to \infty$). For the case shown in Fig. 4 $t_{cr} = 15\ \mu s$.

If $\tau > \tau_{cr}$ the solution with $D = \mathrm{Const}$ is impossible, the compression wave generated in the burn-down zone penetrates through the CJ plane and accelerates the detonation wave smoothly.

If the heat losses from the gas phase dominate for some period of time the function $q(y)$ has an intermediate minimum (curves 2–5 in Fig. 3). Figure 5 illustrates the corresponding particle velocity profiles in the burn-down zone.

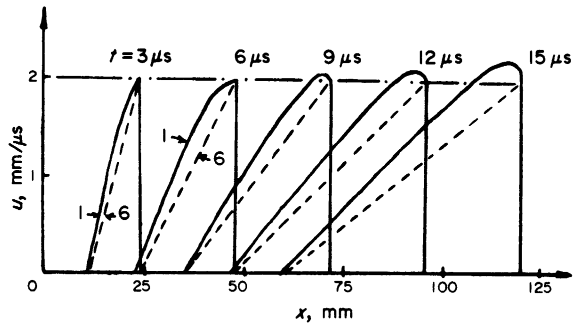

Fig. 4. Dynamics of secondary compression wave evolution in the burn-down zone when the process is exothermic.

For curve 5 $Q_2 < 0$. One can see that at any point of the flow the gas is moving slower than at the CJ plane and the flow remains supersonic everywhere in the burn-down zone. $Q_2 > 0$ for curves 2–4 in Fig. 3. Additional heating the gaseous products results in the secondary compression wave formation, and the flow in some region becomes subsonic. The larger are the burn-down zone heat release Q_2 and the rate of its production the earlier arises the region of the subsonic flow. The gas velocity gradient at the leading edge of such a compression wave approaches infinity asymptotically (at $\tau \to \infty$), that is the detonation wave might propagate at a constant velocity D_{CJ} for an unlimited time period. However there are two causes which must limit this time period (or a dwell time of a steady nonideal detonation wave). The first is the dependence of the exothermic reaction rate on flow parameters, and the second is the gasdynamic instability of the flow.

The behavior of small perturbations of the flow velocity in the burn-down zone δw_1 is determined by the equation

$$\frac{\partial \delta w_1}{\partial \tau} + w_1 \frac{\partial \delta w_1}{\partial y} + \delta w_1 \frac{\partial w_1}{\partial y} = 0 \tag{16}$$

derived from the eqn (9) when substituting $w = w_1 + \delta w_1$ and linearizing eqn (9) with respect to $\delta w_1 / w_1 \ll 1$. Here w_1 is the nonperturbed solution of eqn (9) determined by eqns (12) and (13). At the head of the secondary compression wave $w_i = 0$, and hence

$$|\delta w_1| = \text{Const} \exp\left[-\frac{\partial w_1}{\partial y} \cdot \tau\right]; \frac{\partial w_1}{\partial y} \sim -\frac{\partial u}{\partial y}. \tag{17}$$

Thus disturbances of finite amplitude which may arise locally in the burn-down zone due to the fluctuations in the reaction rate are enhanced on their way to the CJ plane and overtake the latter. The complex consisting of a detonation wave and burn-down zone becomes more unstable if the growth of the secondary compression wave amplitude accelerates the exothermic reaction.

The steady propagation of a nonideal detonation is possible up to the moment

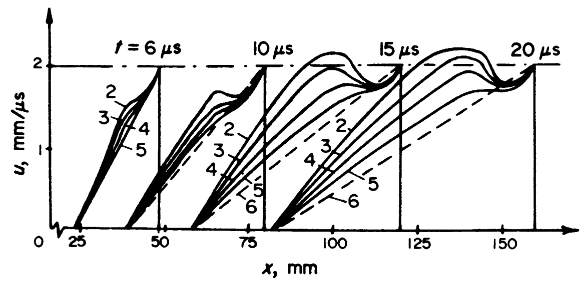

Fig. 5. Dynamics of secondary compression wave evolution when the process is endothermic in some part of the flow, (2–5)dq/dy changes its sign $Q_{2,2} > Q_{2,3} > Q_{2,4} > 0$; $Q_{2,5} < 0$; (6) $Q_2 = 0$, isentropic rearfaction wave.

when the secondary compression wave with infinite pressure gradient arrives at the CJ plane. Before this happens a region of subsonic flow arises in the rearfaction wave. In the case when the endothermic stage in the burn-down zone is absent (see Fig. 3, curve 1) the flow adjacent to the CJ plane becomes subsonic at the moment $\tau = \tau_{cr}/2$ (see eqns 14 and 15).

Estimation of the dwell-time of a steady nonideal detonation wave

The dwell-time of a nonsteady nonideal detonation wave t_d is a sum of two time intervals: t_{cr} is for the secondary shock to be formed and t_2 for the secondary shock to overtake the primary detonation wave. The latter can be evaluated assuming that in the secondary shock wave the chemical energy stored in the substance which enters its front is released immediately. The time interval t_{cr} can be estimated for an arbitrary heat release-time history.

The flow region to be considered is restricted like in the previous section by the straight trajectory of the CJ plane $x = D_{CJ}t$ and by the quiescent piston $x = 0$.

Rearrangement of eqns (1) yields:

$$\frac{2}{c}\frac{\mathrm{d}c}{\mathrm{d}t} - (\gamma - 1)\frac{\mathrm{d}\ln\rho}{\mathrm{d}t} = \frac{\gamma(\gamma-1)}{c^2}\frac{\mathrm{d}Q}{\mathrm{d}t}. \tag{18}$$

Integrating this equation along the piston trajectory and using the initial conditions at the piston: $c(0,\ 0) = D_{CJ}/2$; $p(0,\ 0) = p_r = [\rho_0 D^2_{CJ}/(\gamma+1)] \times [(\gamma+1)/2\gamma]^{2\gamma/(\gamma-1)}$; $Q(0,\ 0) = Q_{CJ}$; $\rho_r(0,\ 0) = 4\gamma p_r/D^2_{CJ}$, where p_r and ρ_r are the pressure and density at the wall in the isentropic rearfaction wave attached to an ideal strong detonation wave (K. P. Stanyukovich, 1971). Thus we obtain the relation between c and ΔQ:

$$c^2(0, t) = (D_{CJ}/2)^2 + \gamma(\gamma-1)\left[\Delta Q - \int_0^t p\,\frac{\mathrm{d}(1/\rho)}{\mathrm{d}t}\,\mathrm{d}t\right] \tag{19}$$

where $\Delta Q = Q(0, t) - Q_{CJ}$.

If the expansion of the fluid which results in the secondary wave generation is due to the reaction occurring in the region adjacent to the piston, the last term in eqn (19) can be approximated as follows:

$$\gamma(\gamma-1)\left[\Delta Q - \int_0^t p\,\frac{\mathrm{d}(1/\rho)}{\mathrm{d}t}\,\mathrm{d}t\right] = \frac{\gamma}{\bar{\gamma}}(\gamma-1)\Delta Q \tag{20}$$

where $1 < \bar{\gamma} < \gamma$. The extremes in this inequality are: γ for case of isobaric expansion of the reacting fluid and 1 for the isochoric process.

After introducing the average $\bar{\gamma}$ substitution of eqn (20) into eqn (19) yields:

$$c(0, t) = \sqrt{\left[(D_{CJ}/2)^2 + \frac{\gamma}{\bar{\gamma}}(\gamma-1)(Q - Q_{CJ})\right]} \tag{21}$$

and

$$M(0, t) = 2/\sqrt{\left[1 + \frac{2\gamma}{\bar{\gamma}(\gamma+1)}(Q/Q_{CJ} - 1)\right]}. \tag{22}$$

If at some moment t_{cr} $c(0, t)$ becomes more than D_{CJ} due to heat evolution nearby the piston, then $M(0, t)$ becomes less than 1. This means that a secondary shock wave must arise at some distance from the piston due to the coalescence of characteristics. This shock wave prevents the formation of subsonic flow at the piston. The upper limit for the time of this secondary shock formation can be estimated using the equation:

$$M(0, t_{cr}) = 1 \text{ or } Q(0, t_{cr}) = Q_{CJ}\left[1 + \frac{3\bar{\gamma}(\gamma+1)}{2\gamma}\right] = (4 \div 7)Q_{CJ}. \tag{23}$$

This criterion of secondary shock formation time is quite different from that derived in Section 2. This is due to the difference in flow parameters in the burn-down zone. Equations (23) represents the case of an initiator with a very short action time, i.e. the case of a very steep rearfaction wave, and the process considered in Section 2 corresponds to a long lasting initiator. If the piston trajectory in the burn-down zone is defined by eqn (6) the time of steady propagation of the detonation wave $t_d = t_{cr} + t_2$ is equal to 2.0 μs and 2.5 μs for energy release profiles 3 and 4 respectively (Fig. 3). If the piston stops at the moment of *CJ* plane formation then t_{cr} only is equal to 16 μs and 23 μs for the same heat release profiles.

Thus the flow expansion downstream of the *CJ* plane strongly affects the time interval required for the secondary shock formation.

Equation (23) and results of the previous paper (Ermolaev *et al.*, 1976) are used to estimate t_{cr} and L_{cr} for LVD in PETN charges confined in slightly deformable tubes. For short acting initiator and LVD propagating at the velocity 3 mm/μs neglecting the heat losses due to the expansion of the confinement in the burn-down zone we obtain $t_{cr} = 60$ μs and $L_{cr} = 300$ mm. Since the t_{cr} is an upper estimated value the above numbers should be considered as approximate ones but of the right order.

References

Bdzil, J. B. (1976) Perturbation methods applied to problems in detonation physics, Prep. 6th *Symp. on Detonation*, San Diego, California, 24–27.

Chaiken, R. F. and Edwards, J. C. (1976) A kinetic lattice approach to detonation in heterogeneous explosives. *Prep.* 6th *Sympos. (Int.) on Detonation*, San Diego, California, 347–353.

Dremin, A. N., Savrov, S. D., Trofimov, V. S. and Shvedov, K. K. (1970) *Detonation Waves in Condensed Media*, Izdatel'stvo Nauka, Moscow.

Ermolaev, B .S., Khasainov, B. A., Borisov, A. A. and Korotkov, A. I. (1976) Low velocity detonations in cast and pressed high explosives, *Acta Astronautica* 3(11/12), 1033–1047.

Kuznetsov, N. M. (1968) On nonuniqueness and stability of detonation regimes in a bounded medium, *U.S.S.R. J. Prikladnaya Mekhanika i tekhnicheskaya Fiz.*, No. 1, 45–56.

Stanyukovich, K. P. (1971) *Unsteady Motion of Continuous Media*. Izdatel'stvo Nauka, Moscow.

Sternberg, H. M. (1970) Similarity solutions for reactive shock waves. *Quart. J. Mech. Appl. Math.* XXIII, Part 1, 77–79.

Appendix

Nomenclature

- $c = \sqrt{(\gamma p/\rho)}$ sonic velocity
- D shock or detonation wave velocity
- $E = p/(\gamma - 1)\rho$ internal energy
- $M = (D - u)/c$ flow Mach number
- l thickness of the primary detonation zone
- L_{cr} steady propagation distance of the nonideal detonation
- p pressure
- $p_{CJ} = \rho_0 D_{CJ}^2/(\gamma + 1)$ pressure at the *CJ* plane
- $\tilde{p} = p/p_{CJ}$ reduced pressure
- Q energy evolved in the reacting flow
- Q_{CJ} energy released at the *CJ* plane
- Q_∞ final net heat release
- $Q_2 = Q_\infty - Q_{CJ}$ heat release in the burn-down zone
- t time
- t_{cr} time to a secondary shock wave formation
- t_d dwell time of a steady nonideal detonation wave
- t_2 time required for the secondary shock to overtake the detonation wave
- t_1 Lagrangian label for a particle path
- u particle velocity
- $u_{CJ} = D_{CJ}/(\gamma + 1)$ particle velocity at the *CJ* plane
- w_1 nondimensional particle velocity, eqn (9)
- x space coordinate
- x_p piston trajectory
- y dimensionless space coordinate
- γ specific heat ratio C_p/C_v
- $\epsilon^2 = Q_2/Q_{CJ}$ small parameter
- ρ density
- τ reduced time, eqn (9)