

# PERIOD DOUBLING AND TRANSITION TO CHAOS IN PULSED COMBUSTION MODES OF REACTIVE SOLIDS

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**Abstract:** Stationary combustion waves in reactive solids are known to propagate only within a specific range of parameters. The present study investigates the pulsed combustion of reactive solids under conditions where the combustion front propagating through the solid phase becomes unstable. Numerical methods were used to determine how the instantaneous front velocity changes over time. The developed model demonstrated that a slight departure from the stability threshold of the steady-state mode resulted in periodic oscillations of the front velocity following an initial induction period. Within a specific parameter range, the emergence of two-periodic, four-periodic, and more complex modes, resembling the well-known period-doubling mechanism in basic discrete-time dynamic systems, were observed. The boundaries of the parameter ranges that lead to oscillations with one, two, and four periods were identified. Further deviation from the stability boundary can lead to less understood modes where the periodicity changes, potentially resembling strange attractors observed in the theory of dynamic systems. It is demonstrated that analyzing the frequency spectrum of oscillations is crucial for understanding these modes. The oscillation spectra for the studied modes, revealing the characteristic frequencies of front pulsations are presented. The challenges in numerically modeling these modes, including the significant impact of the grid step and rounding accuracy on the calculation results, are discussed. Also, the need for extended calculation times to accurately capture the effects of front initiation by a hot surface and to reliably identify the different regular modes is highlighted.

**Keywords:** combustion theory; numerical combustion modeling; pulsed modes; period doubling; strange attractor; transition to chaos

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## Figure Captions

**Figure 1** Front velocity  $u_f$  vs. time  $t$  (a) and spectrum of oscillations (b) at  $\gamma = 0.14$

**Figure 2** Front velocity  $u_f$  vs. time  $t$  (a) and spectrum of oscillations (b) at  $\gamma = 0.115$

**Figure 3** Front velocity  $u_f$  vs. time  $t$  (a) and spectrum of oscillations (b) at  $\gamma = 0.064$

**Figure 4** Front velocity  $u_f$  vs. time  $t$  at  $\gamma = 0.058$

**Figure 5** Front velocity  $u_f$  vs. time  $t$  (a) and spectrum of oscillations (b) at  $\gamma = 0.048$

## Table Caption

Front velocity for steady propagation modes

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