WELL-POSEDNESS OF NONISOTHERMAL EULER MODELS OF TWO-PHASE FLOWS

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Abstract: A well-posed mathematical model of nonisothermal two-velocity two-temperature two-phase flow of bubbly liquid has been proposed. The model is based on two-phase Euler equations with the introduction of an additional pressure at the gas bubble surface, which ensures the well-posedness of the Cauchy problem for a system of governing equations with homogeneous initial conditions. The additional pressure is obtained using the local pressure coefficient averaged over the phase interface for different Reynolds numbers of relative motion of phases. The applicability conditions of the model have been formulated. The model has been validated for a one-dimensional problem of shock wave propagation in water with air bubbles with a volume gas content of 0.5% to 30% by comparing the calculation results with the experimental data. It has been shown that the model provides satisfactory results for the shock propagation velocity and the shock-induced motion of gas bubbles in bubbly liquid at volume gas content above 2%.

Keywords: two-phase flow; Euler equations; Cauchy problem; bubbly liquid; shock wave; comparison with experiment

Acknowledgments

The work was supported by the Russian Ministry of Education under the state contract No. 14.609.21.0001 (contract ID RFMEFI57914X0038) "Development of technology for the creation of hydrojet thrust in high-speed vehicles and the establishment of stand demonstrator of hydrojet pulse-detonation engine" in the framework of the Federal Target Program "Research and development in priority areas of Russian scientific and technological complex for 2014–2020."

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Received December 18, 2015

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